Simplex Algorithm
CSCI 532

Test 1 Logistics

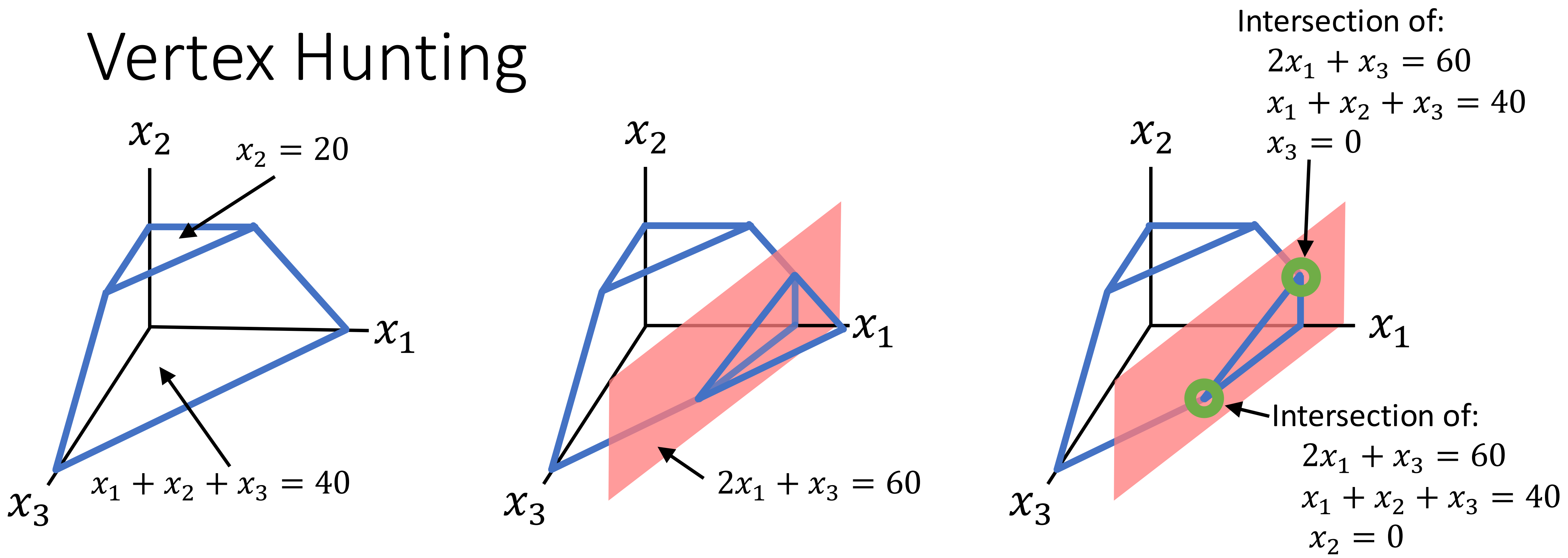
1. During class on Thursday 3/12.
2. You can bring your book and any notes you would like, but no electronic devices.
3. You may assume anything proven in class or on homework.
4. Three questions (13 points):
   1) Flow Network/Min Cut application (5 points).
   2) Linear Program application (5 points).
   3) Conceptual question (3 points).

Simplex Algorithm

```
Simplex(LP)
   v = vertex in feasible region of LP
   while ∃ neighbor v’ with better objective value
      v = v’
   return v
```

How do we find vertices?

Vertex Hunting



Definition: Two vertices are *neighbors* if they share $n - 1$ defining inequalities.

Plan: Move from vertex to vertex by following line formed by intersection of $n - 1$ inequalities.

Simplex Algorithm

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   return v
```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Step 1: Check if current vertex is optimal.

Let $x = (x_1, \ldots, x_n)$. Feasible origin ($x = (0, \ldots, 0)$) $\Longrightarrow$ vertex, because?

Suppose origin is feasible.

Simplex Algorithm

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Step 1: Check if current vertex is optimal.

Let $x = (x_1, \dots, x_n)$. Feasible origin ($x = (0, \dots, 0)$) $\Longrightarrow$ vertex, because it uniquely satisfies n constraints ($x \geq 0$).

Simplex Algorithm

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Simplex(LP)
   v = vertex in feasible region of LP
   while ∃ neighbor v' with better objective value
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```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Step 1: Check if current vertex is optimal.

How can we tell if the origin is optimal?

$$\max 12x_1 + 32x_2 - 9x_3 - 2x_4$$

Simplex Algorithm

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   while ∃ neighbor v' with better objective value
      v = v'
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```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Step 1: Check if current vertex is optimal.

Origin is optimal $\Leftrightarrow c_i \leq 0$, for all i:

$$\mathbf{max} -12\boldsymbol{x_1} - 32\boldsymbol{x_2} - 9\boldsymbol{x_3} - 2\boldsymbol{x_4}$$

Simplex Algorithm

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Simplex(LP)
   v = vertex in feasible region of LP
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```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Step 1: Check if current vertex is optimal.

Origin is optimal $\Leftrightarrow c_i \leq 0$, for all i:

If origin is optimal, increasing x_i, for any i will decrease objective

$$\Longrightarrow c_i x_i \geq c_i(x_i + \varepsilon) \Longrightarrow c_i 0 \geq c_i(0 + \varepsilon) \Longrightarrow c_i \leq 0$$

$$\mathbf{max} -12x_1 - 32x_2 - 9x_3 - 2x_4$$

Simplex Algorithm

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$$\Longrightarrow c_i x_i \geq c_i(x_i + \varepsilon) \Longrightarrow c_i 0 \geq c_i(0 + \varepsilon) \Longrightarrow c_i \leq 0$$

if $c_i \leq 0$, for all i, $c_i(x_i + \varepsilon) \leq c_i x_i$, i.e., lower objective value

$$\boxed{\mathbf{max} -12x_1 - 32x_2 - 9x_3 - 2x_4}$$

Simplex Algorithm

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   v = vertex in feasible region
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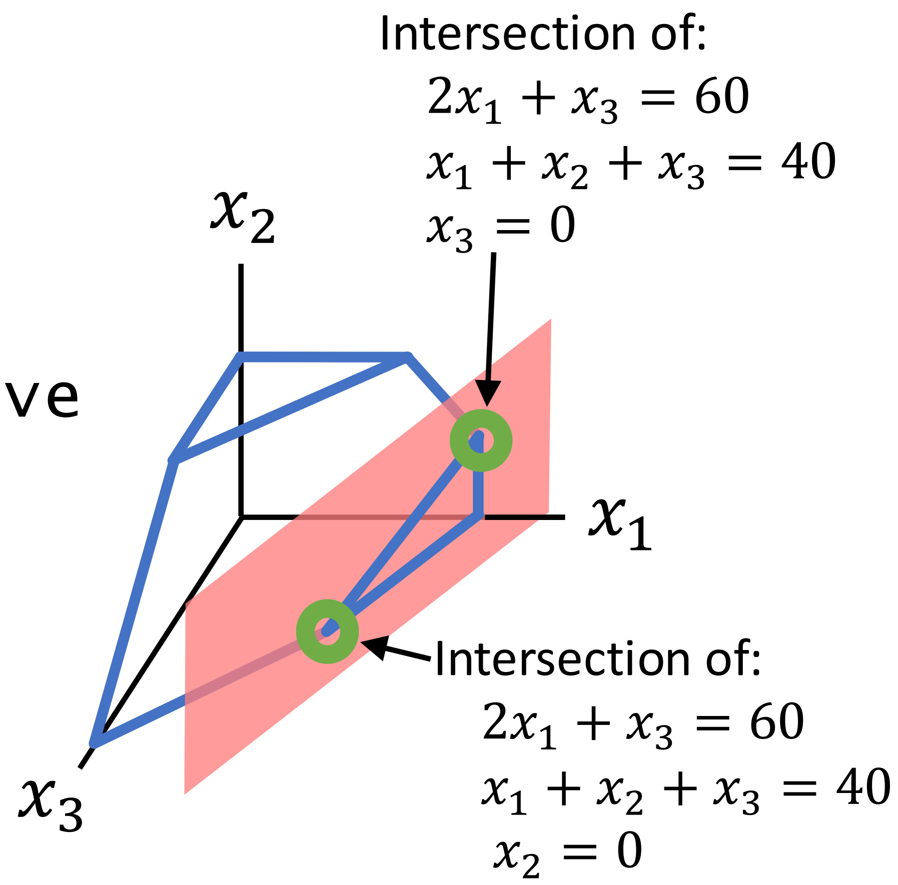

Step 2: Move to a neighboring vertex.

Simplex Algorithm

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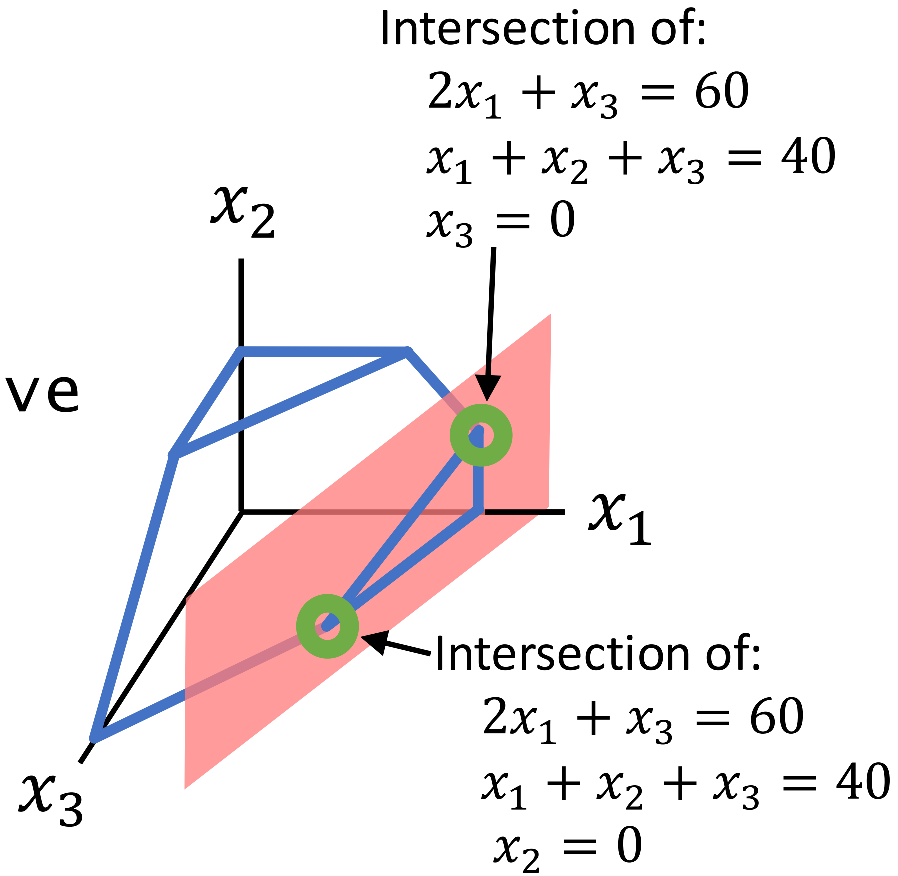
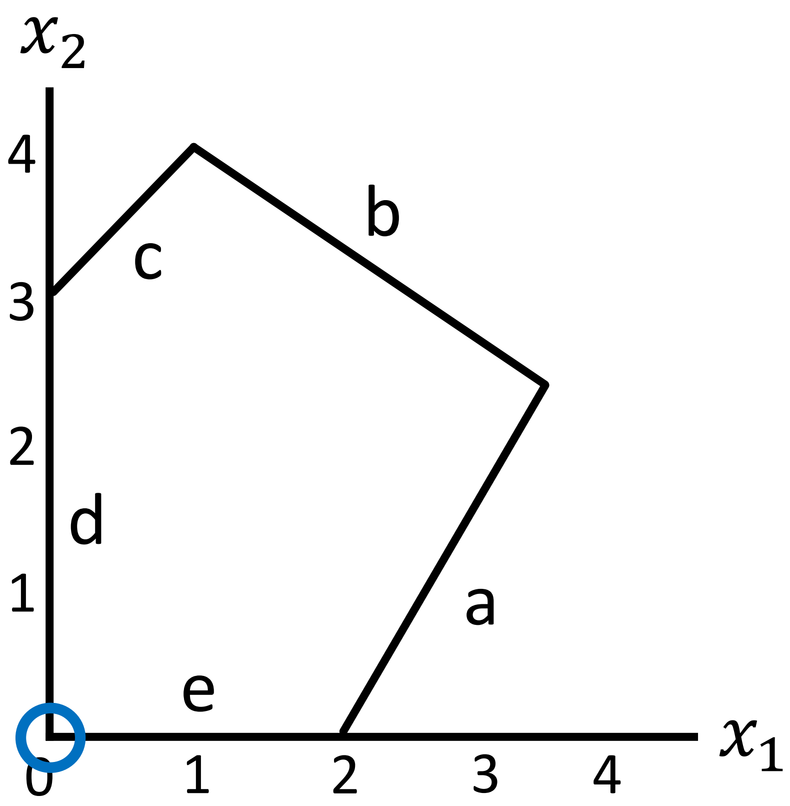


Step 2: Move to a neighboring vertex.

Relax any constraint that allows us to increase the objective function (i.e. x_i, where $c_i > 0$). Keep increasing x_i until another constraint becomes tight.

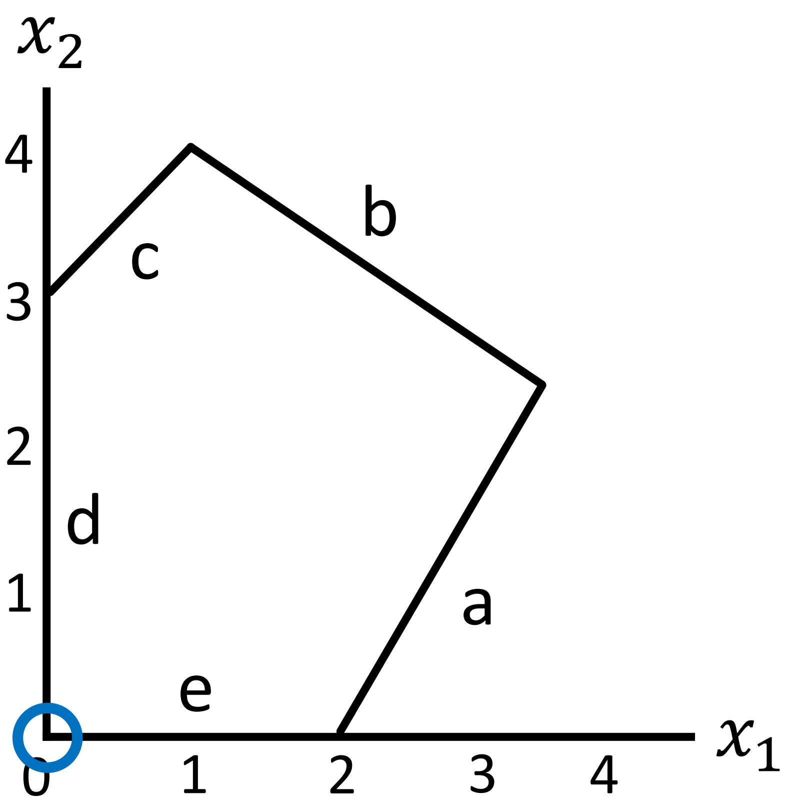
Simplex Algorithm



Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

1. Start at origin, $(x_1, x_2) = (0,0)$.
2. Relax a tight constraint.
3. Stop when another constraint is met.
4. New vertex = intersection of new constraint.

Simplex Algorithm



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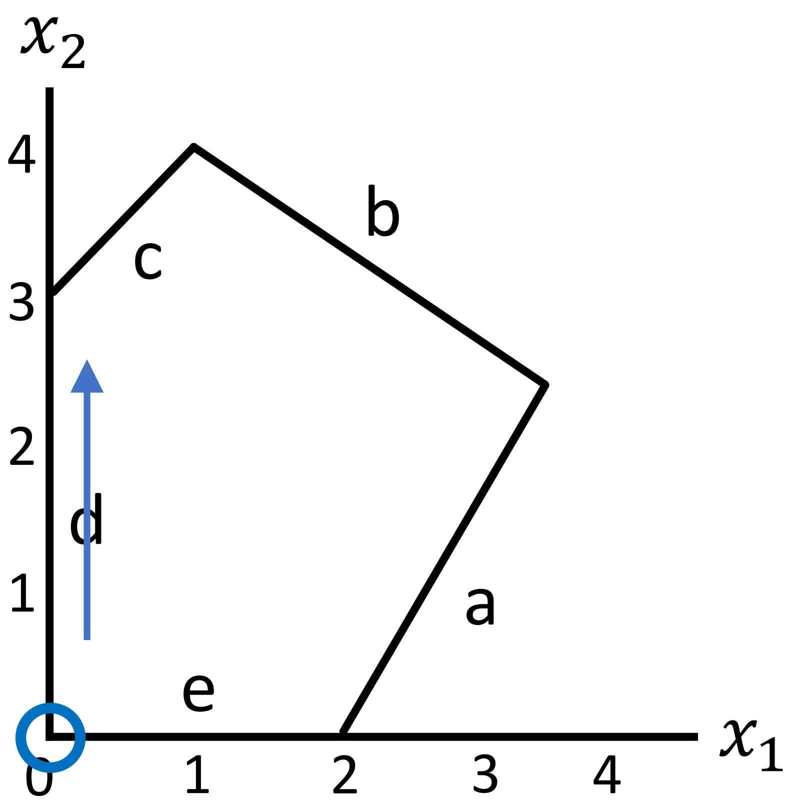
1. Start at origin, $(x_1, x_2) = (0,0)$.
2. Relax a tight constraint.
   Either d or e. Suppose e.
3. Stop when another constraint is met.

4. New vertex = intersection of new constraint.

Simplex Algorithm



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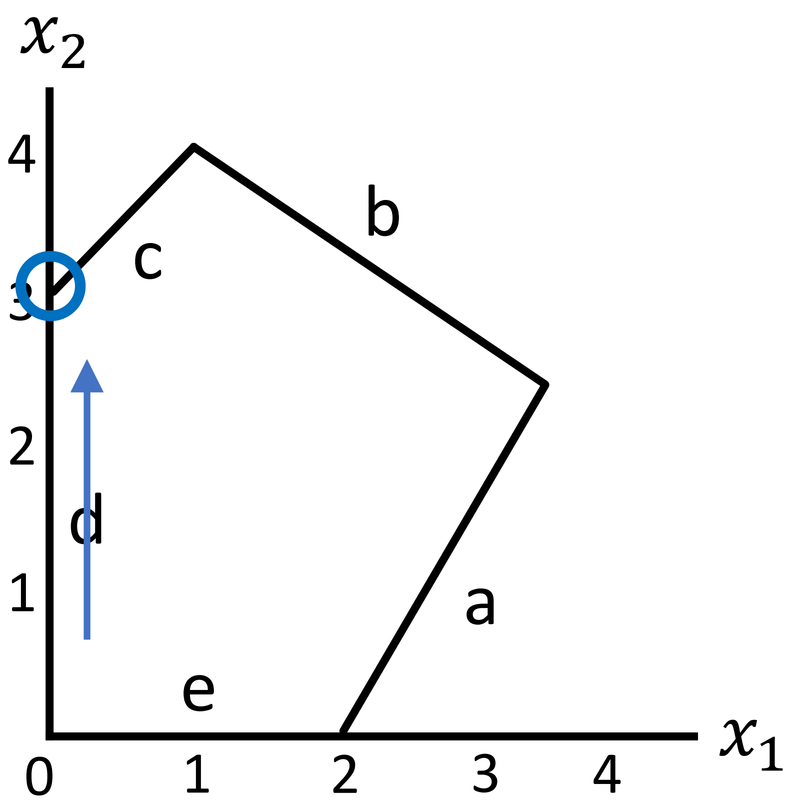
1. Start at origin, $(x_1, x_2) = (0,0)$.
2. Relax a tight constraint.
   Either d or e. Suppose e.
3. Stop when another constraint is met.
   Let x_2 increase until another constraint is met (a – never, b – 4.5, c – 3)
4. New vertex = intersection of new constraint.

Simplex Algorithm



Objective: $\max 2x_1 + 5x_2$

Subject to:

$2x_1 - x_2 \leq 4$ (a)

$x_1 + 2x_2 \leq 9$ (b)

$-x_1 + x_2 \leq 3$ (c)

$x_1 \geq 0$ (d)

$x_2 \geq 0$ (e)

1. Start at origin, $(x_1, x_2) = (0,0)$.
2. Relax a tight constraint.
   Either d or e. Suppose e.
3. Stop when another constraint is met.
   Let x_2 increase until another constraint is met (a – never, b – 4.5, c – 3)
4. New vertex = intersection of new constraint.
   u = intersection of d and c.

Simplex Algorithm

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   return v
```

Objective:	$\max c^T x$
Subject to:	$A x \leq b$
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Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Super easy to do if starting at origin!

Simplex Algorithm

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   v = vertex in feasible region
   while ∃ neighbor v' with > objective
      v = v'
   return v
```

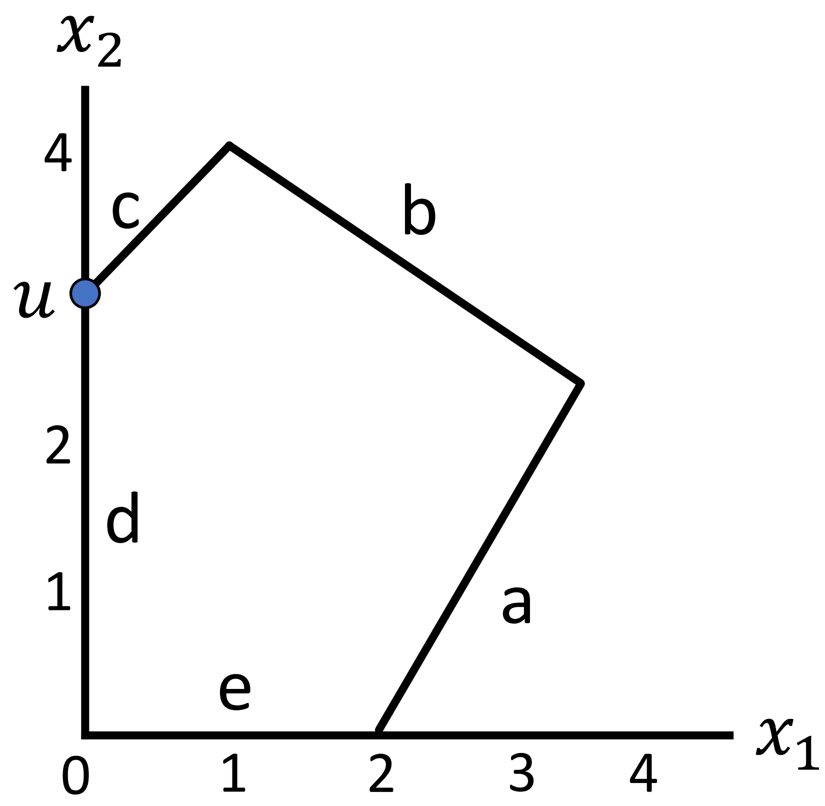

Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

What should we do if we are not at the origin?

Simplex Algorithm

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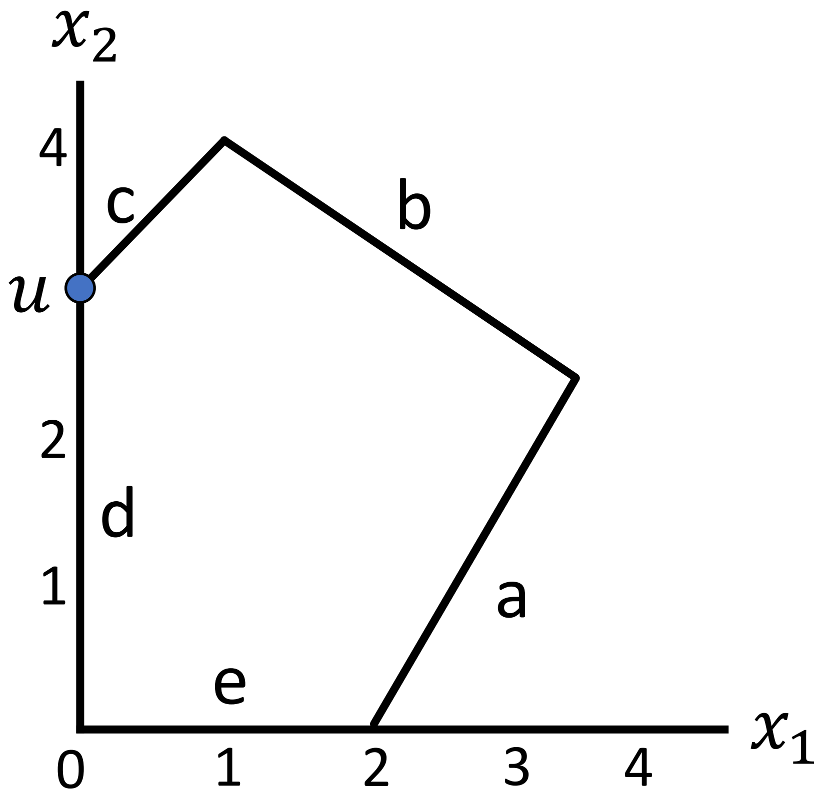
      v = v'
   return v
```



Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Step 3: Redefine coordinate system so u is the origin.

Simplex Algorithm

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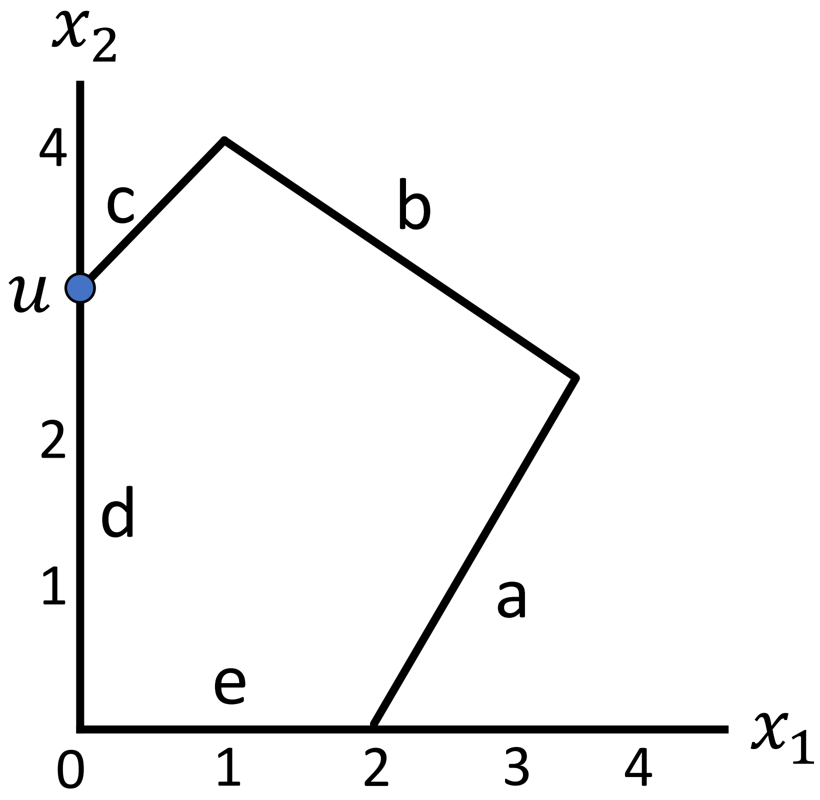

Step 1: Check if current vertex is optimal.

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Step 3: Redefine coordinate system so u is the origin.

For constraint $a_i\mathrm{x} \leq b_i$, $y_i = b_i - a_i\mathrm{x}$

distance from x to constraint *i*

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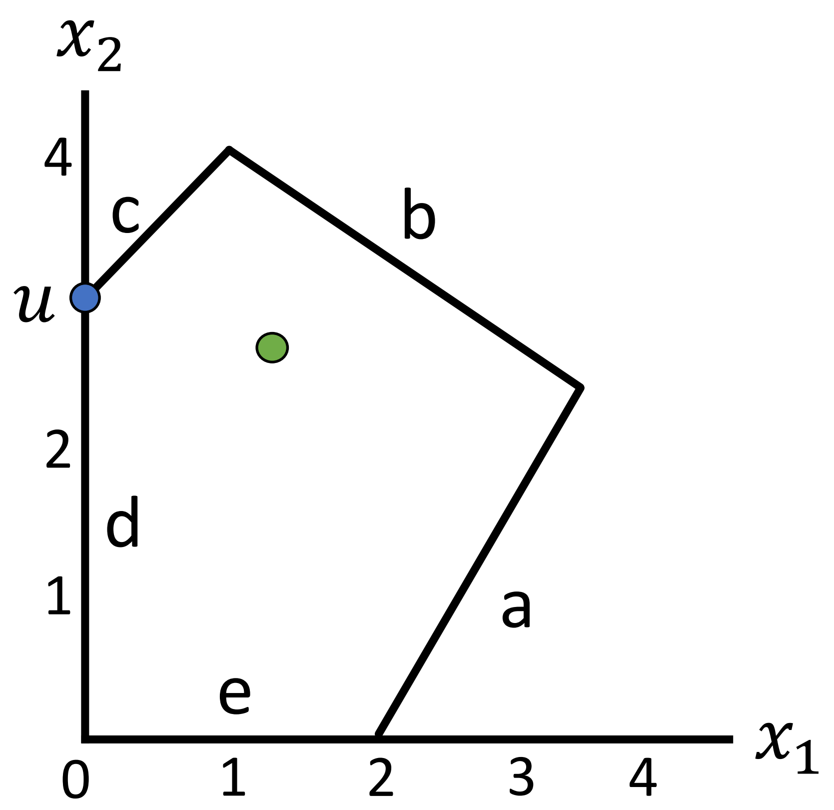

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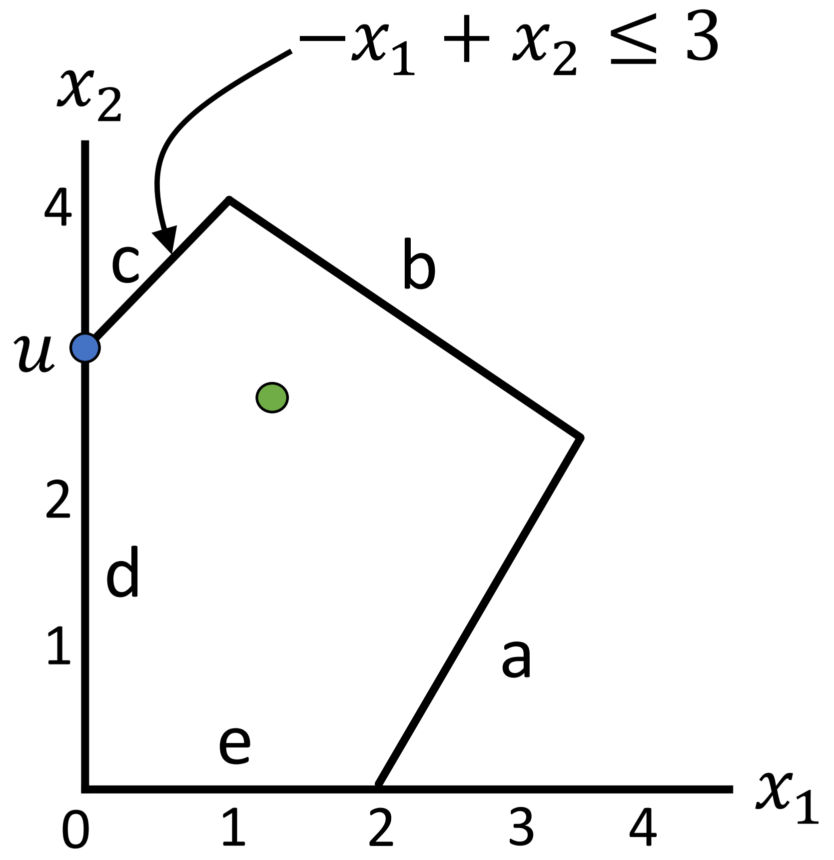

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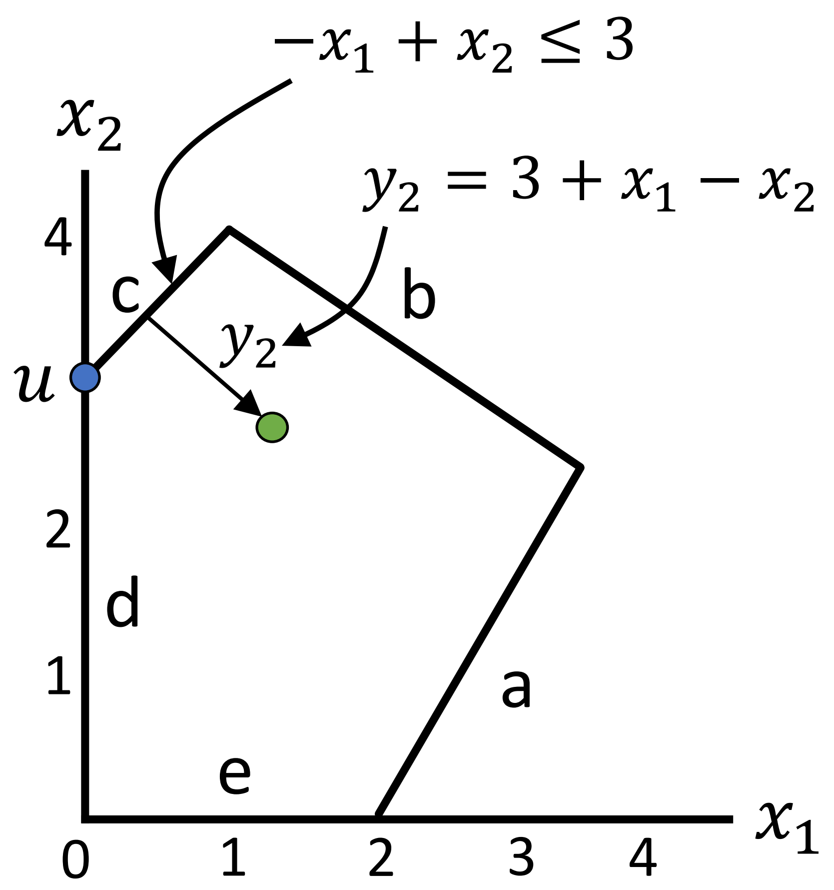

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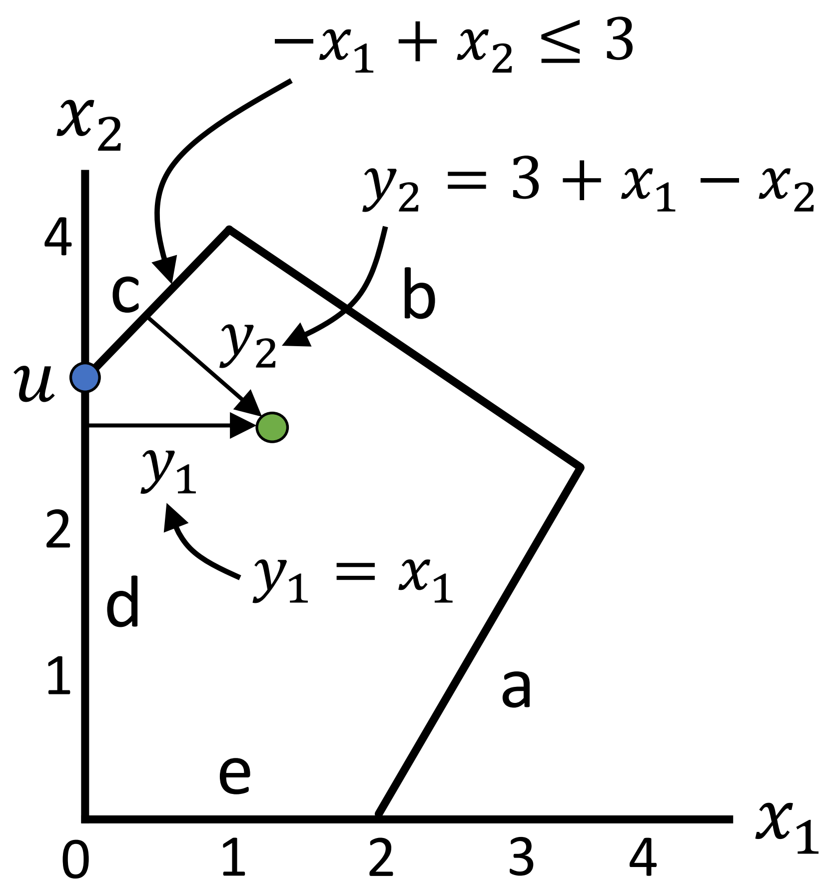

Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Step 3: Redefine coordinate system so u is the origin.

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distance from x to constraint *i*

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```
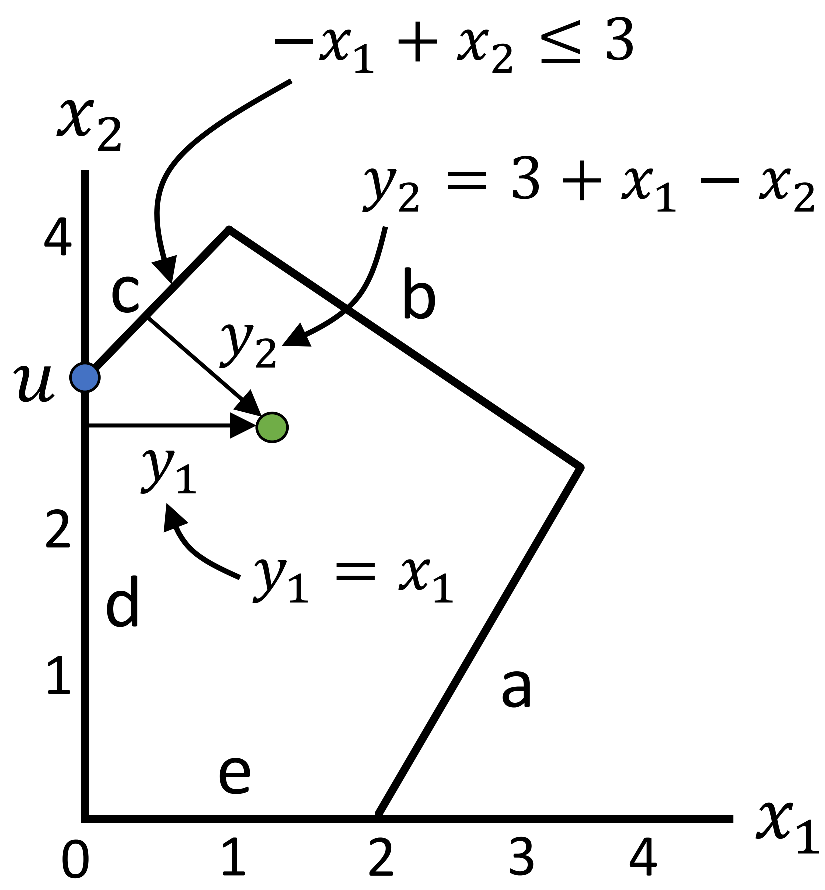

$$(x_1, x_2) = (0,3) \Rightarrow (y_1, y_2) = (0,0)$$



Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Step 3: Redefine coordinate system so u is the origin.

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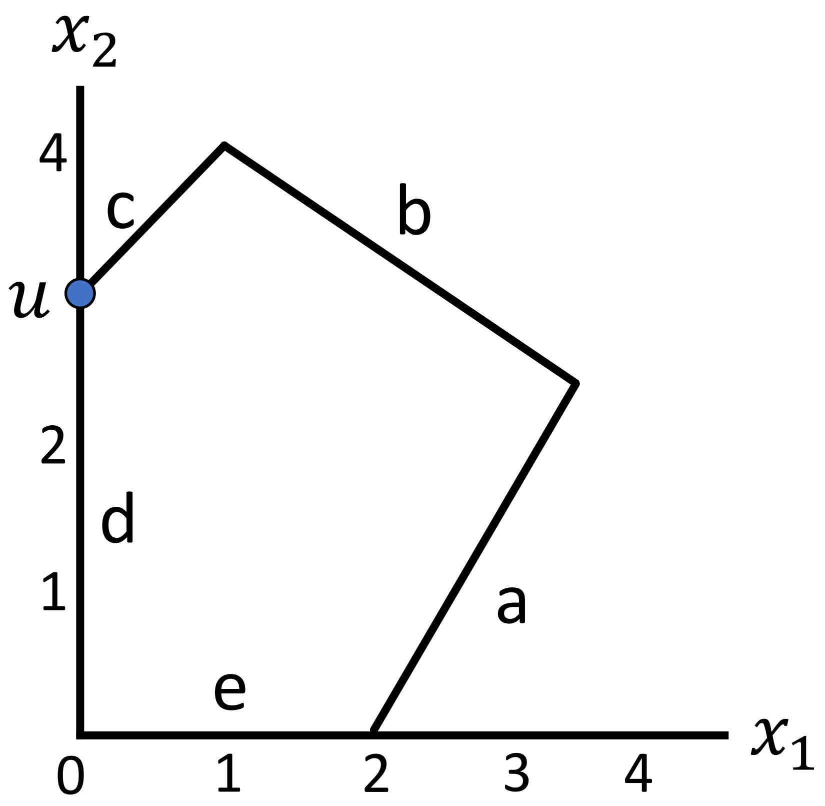

Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Step 3: Redefine coordinate system so u is the origin.

Step 4: ???

Simplex Algorithm

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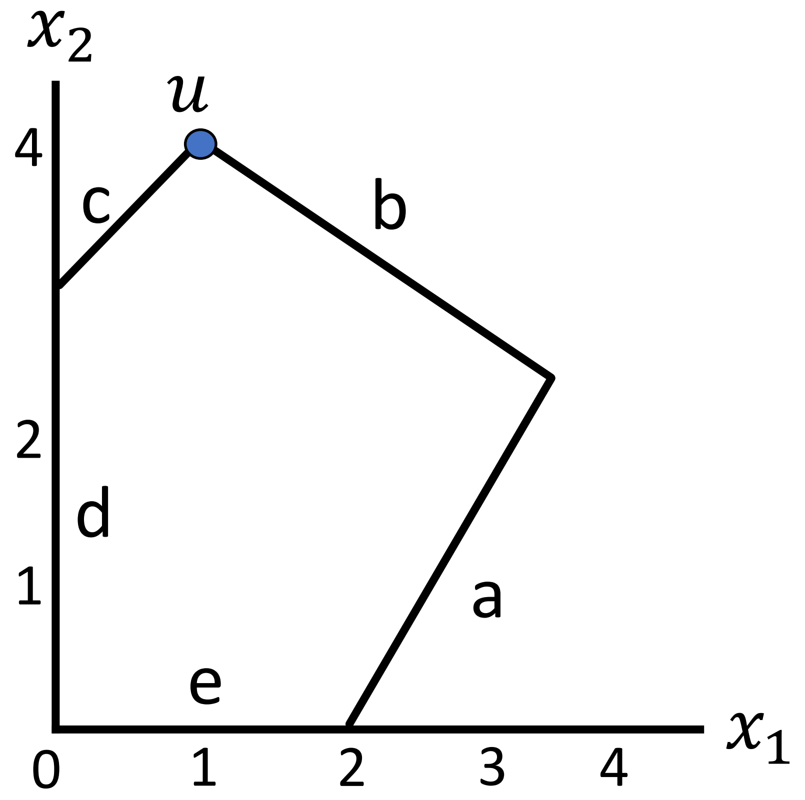

Step 1: Check if current vertex is optimal.

Step 2: Move to a neighboring vertex.

Step 3: Redefine coordinate system so u is the origin.

Step 4: Repeat.

```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
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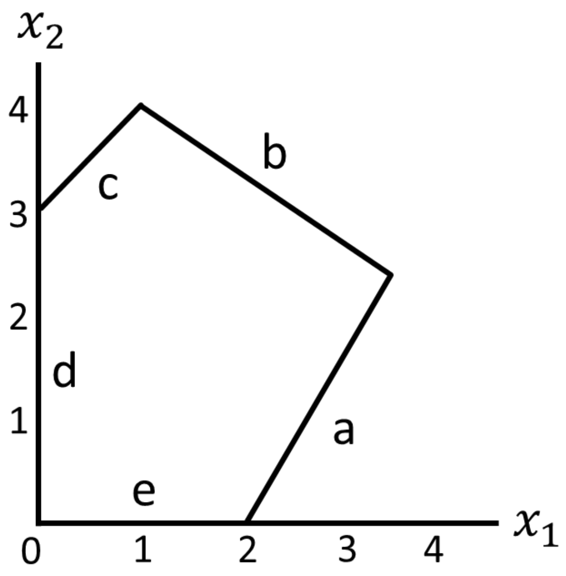

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```

Vertex:

Objective Value:

Relax:

Stop at:

Vertex Local Coordinates:

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Subject to:	$2x_1 - x_2 \leq 4$	(a)
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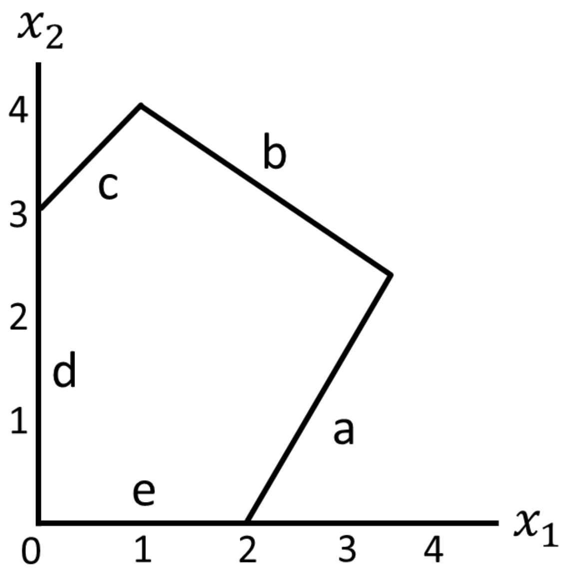

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```

Vertex: Origin (d,e)

Objective Value:

Relax:

Stop at:

Vertex Local Coordinates:

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
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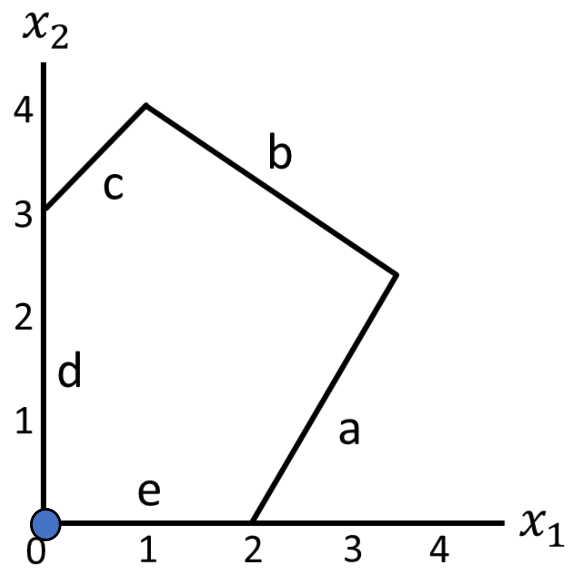

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   return v
```

Vertex: Origin (d,e)

Objective Value: 0

Relax:

Stop at:

Vertex Local Coordinates:

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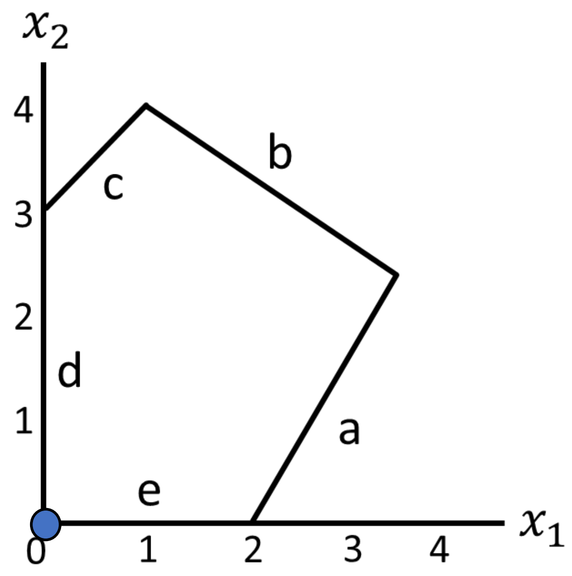

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Vertex: Origin (d,e)

Objective Value: 0

Relax: e

Stop at:

Vertex Local Coordinates:

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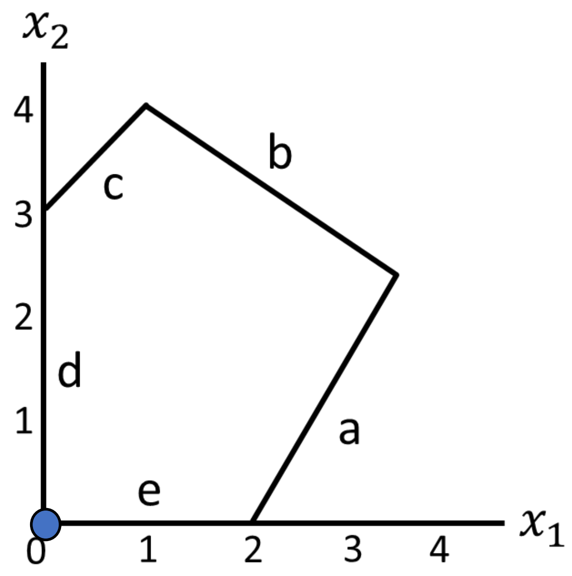

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      v = new intersection.
      for constraint ai·x ≤ bi
         yi = bi - ai·x
      reformulate LP in terms of yi
   return v
```

Vertex: Origin (d,e)

Objective Value: 0

Relax: e

Stop at:

Vertex Local Coordinates:

Objective: $\max 2x_1 + 5x_2$

Subject to:

$2x_1 - x_2 \leq 4$ (a)

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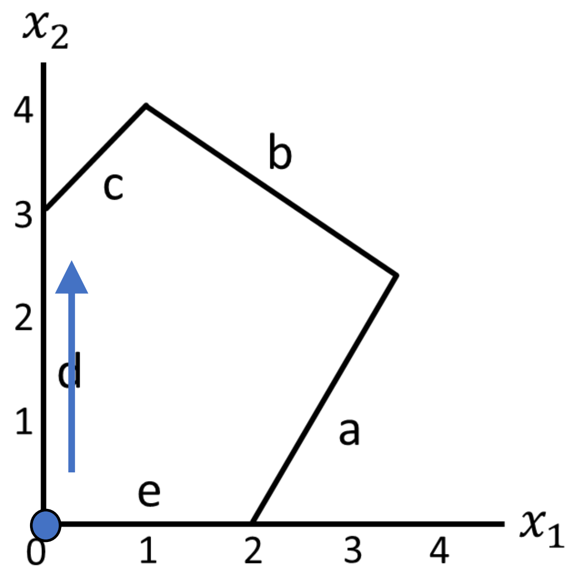

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```

Vertex: Origin (d,e)

Objective Value: 0

Relax: e

Stop at: c,d

Vertex Local Coordinates:

Objective: $\max 2x_1 + 5x_2$

Subject to:

$2x_1 - x_2 \leq 4$ (a)

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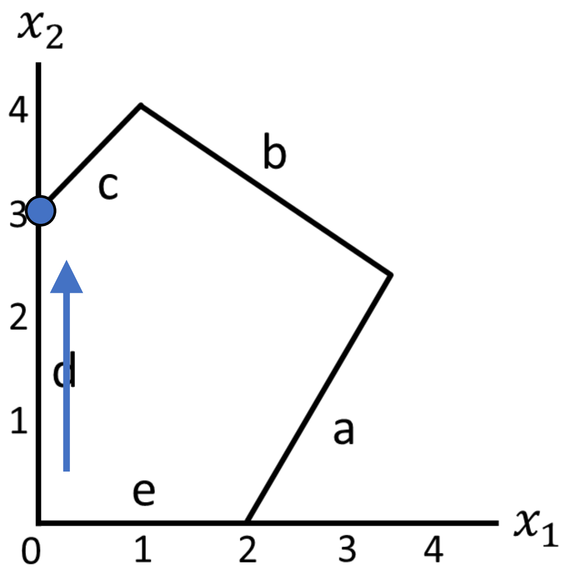

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Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

Relax: e

Stop at: c,d

Vertex Local Coordinates:

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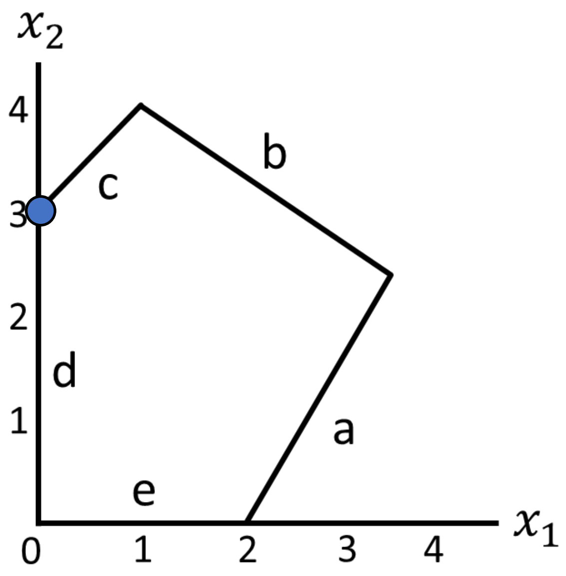

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      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

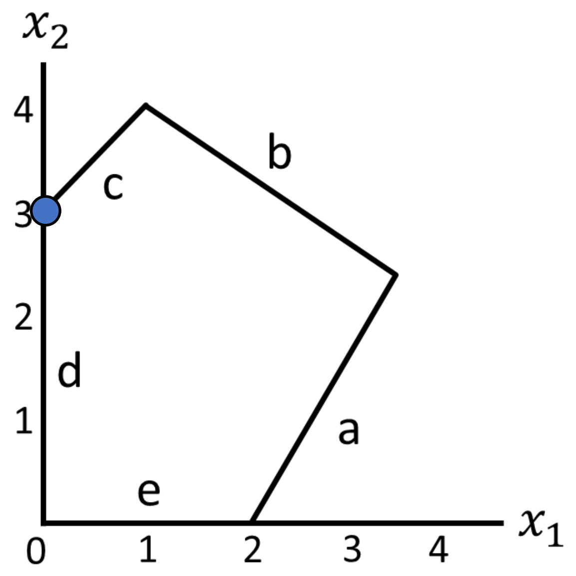
Relax: e

Stop at: c,d

Vertex Local Coordinates:

$$x_1 \geq 0 \Longrightarrow -x_1 \leq 0 \Longrightarrow y_1 = x_1$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

Relax: e

Stop at: c,d

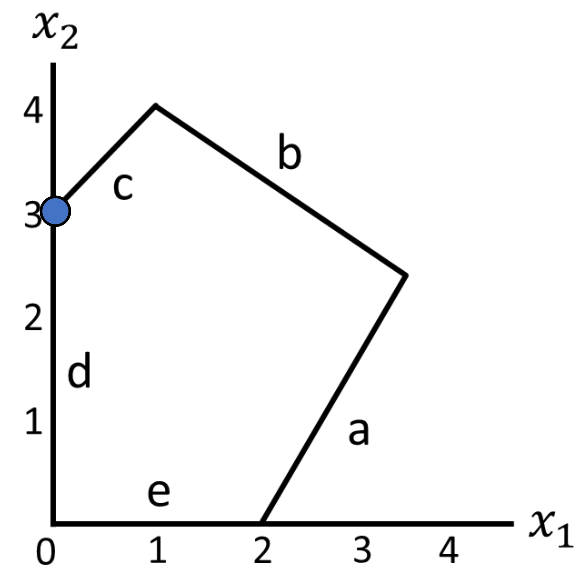
Vertex Local Coordinates:

$$x_1 \geq 0 \Longrightarrow -x_1 \leq 0 \Longrightarrow y_1 = x_1$$

$$-x_1 + x_2 \leq 3 \Longrightarrow y_2 = 3 + x_1 - x_2$$

Objective: $\max 2x_1 + 5x_2$

Subject to:
- $2x_1 - x_2 \leq 4$ (a)
- $x_1 + 2x_2 \leq 9$ (b)
- $-x_1 + x_2 \leq 3$ (c)
- $x_1 \geq 0$ (d)
- $x_2 \geq 0$ (e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

Relax: e

Stop at: c,d

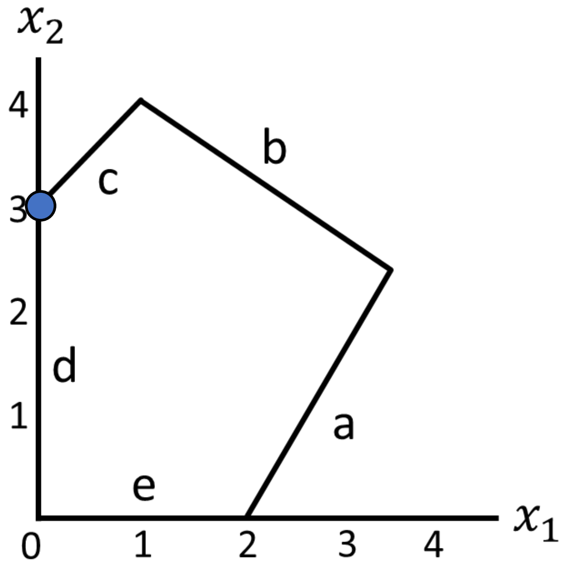
Vertex Local Coordinates:

$$x_1 \geq 0 \Longrightarrow -x_1 \leq 0 \Longrightarrow y_1 = x_1$$

$$-x_1 + x_2 \leq 3 \Longrightarrow y_2 = 3 + x_1 - x_2$$

$$\Longrightarrow x_1 = y_1, x_2 = 3 + y_1 - y_2$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

Relax: e

Stop at: c,d

Vertex Local Coordinates:

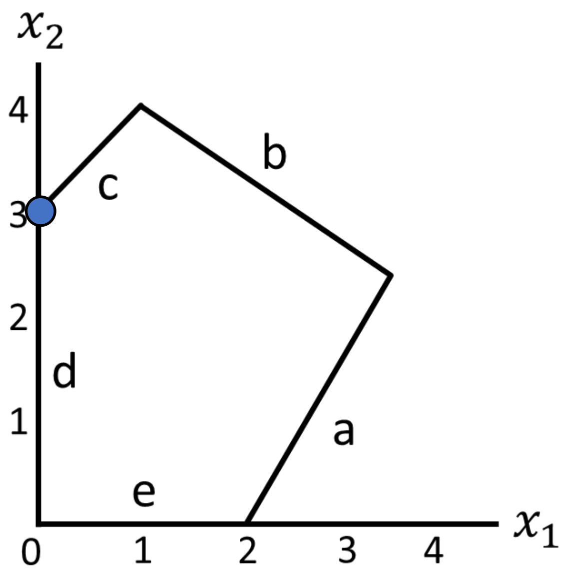
$$x_1 \geq 0 \Longrightarrow -x_1 \leq 0 \Longrightarrow y_1 = x_1$$

$$-x_1 + x_2 \leq 3 \Longrightarrow y_2 = 3 + x_1 - x_2$$

$$\Longrightarrow x_1 = y_1, x_2 = 3 + y_1 - y_2$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

Objective:	max	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



```
Simplified_Simplex(LP)
   v = origin
   while cj > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint ai·x ≤ bi
         yi = bi - ai·x
      reformulate LP in terms of yi
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: 0

Relax: e

Stop at: c,d

Vertex Local Coordinates:

$$x_1 \geq 0 \Rightarrow -x_1 \leq 0 \Rightarrow y_1 = x_1$$

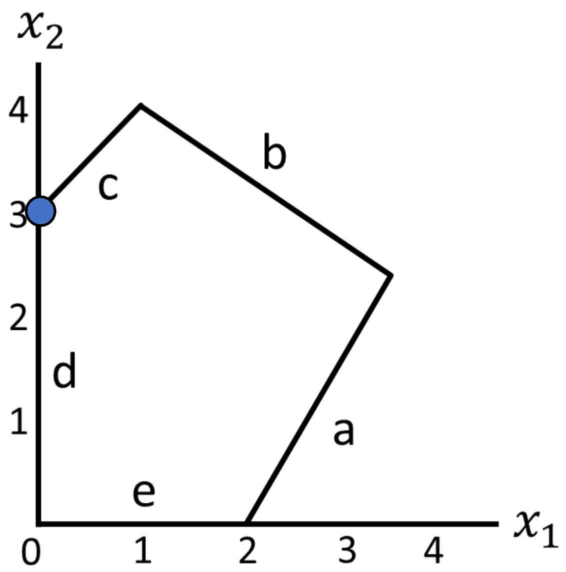
$$-x_1 + x_2 \leq 3 \Rightarrow y_2 = 3 + x_1 - x_2$$

$$\Rightarrow x_1 = y_1, x_2 = 3 + y_1 - y_2$$

$$2x_1 + 5x_2 = 2y_1 + 15 + 5y_1 - 5y_2 = 15 + 7y_1 - 5y_2$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: ~~0~~ 15

Relax: e

Stop at: c,d

Vertex Local Coordinates:

$$x_1 \geq 0 \Rightarrow -x_1 \leq 0 \Rightarrow y_1 = x_1$$

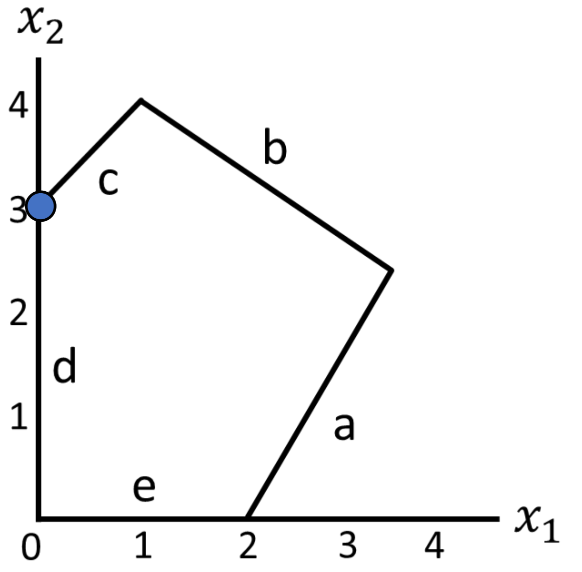
$$-x_1 + x_2 \leq 3 \Rightarrow y_2 = 3 + x_1 - x_2$$

$$\Rightarrow x_1 = y_1, x_2 = 3 + y_1 - y_2$$

$$2x_1 + 5x_2 = 2y_1 + 15 + 5y_1 - 5y_2 = 15 + 7y_1 - 5y_2$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: ~~0~~ 15

Relax: e

Stop at: c,d

Vertex Local Coordinates:

$$x_1 \geq 0 \Rightarrow -x_1 \leq 0 \Rightarrow y_1 = x_1$$

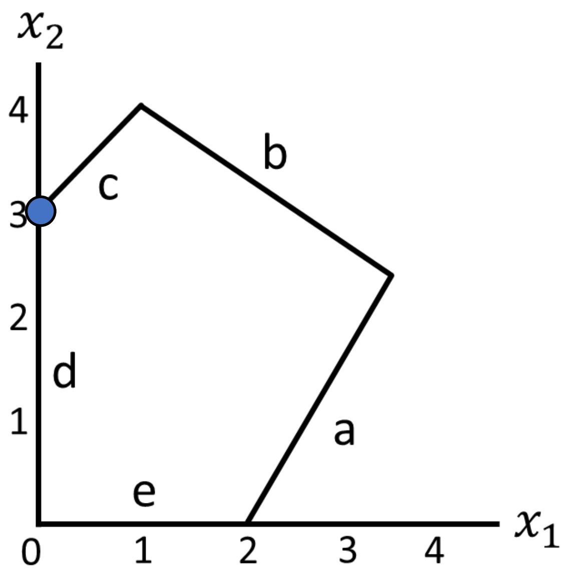
$$-x_1 + x_2 \leq 3 \Rightarrow y_2 = 3 + x_1 - x_2$$

$$\Rightarrow x_1 = y_1, x_2 = 3 + y_1 - y_2$$

$$2x_1 - x_2 = 2y_1 - 3 - y_1 + y_2 = y_1 + y_2 - 3$$

Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
		(b)
		(c)
		(d)
		(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
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      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~Origin (d,e)~~ c,d

Objective Value: ~~0~~ 15

Relax: e

Stop at: c,d

Vertex Local Coordinates:

$$x_1 \geq 0 \Rightarrow -x_1 \leq 0 \Rightarrow y_1 = x_1$$

$$-x_1 + x_2 \leq 3 \Rightarrow y_2 = 3 + x_1 - x_2$$

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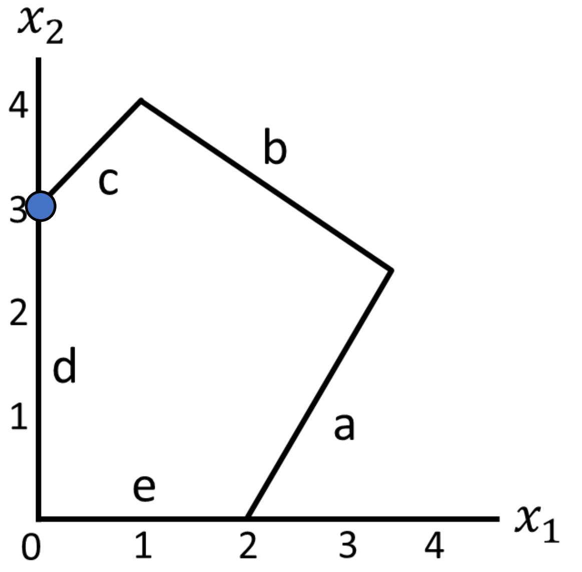
Objective:	$\max 2x_1 + 5x_2$	
Subject to:	$2x_1 - x_2 \leq 4$	(a)
	$x_1 + 2x_2 \leq 9$	(b)
	$-x_1 + x_2 \leq 3$	(c)
	$x_1 \geq 0$	(d)
	$x_2 \geq 0$	(e)

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: c,d

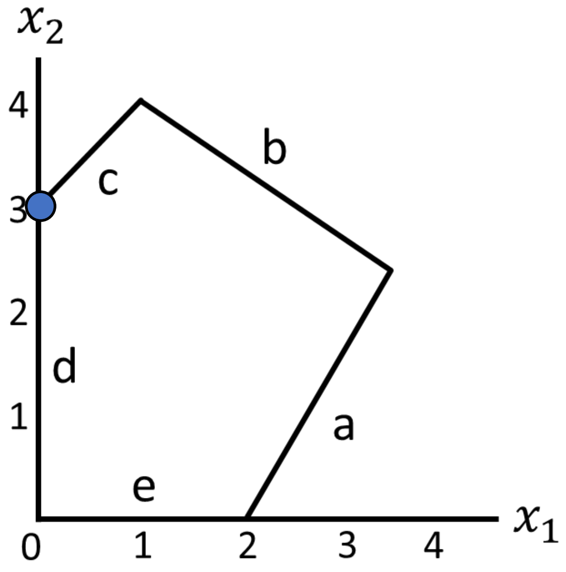
Objective Value: 15

Relax:

Stop at:

Vertex Local Coordinates:

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: c,d

Objective Value: 15

Relax:

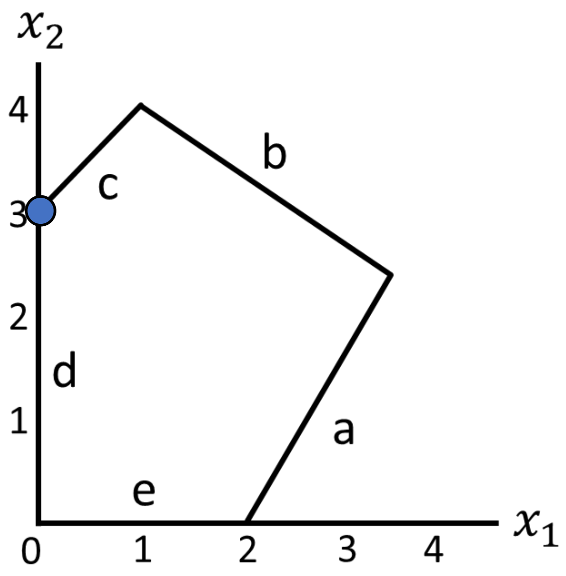
Stop at:

Vertex Local Coordinates:

Which constraint should we relax?

Objective: $\max 15 + 7y_1 - 5y_2$

Subject to:
$y_1 + y_2 \leq 7$ (a)
$3y_1 - 2y_2 \leq 3$ (b)
$y_2 \geq 0$ (c)
$y_1 \geq 0$ (d)
$-y_1 + y_2 \leq 3$ (e)



```
Simplified_Simplex(LP)
   v = origin
   while cj > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint ai·x ≤ bi
         yi = bi – ai·x
      reformulate LP in terms of yi
   return v
```

Vertex: c,d

Objective Value: 15

Relax: d

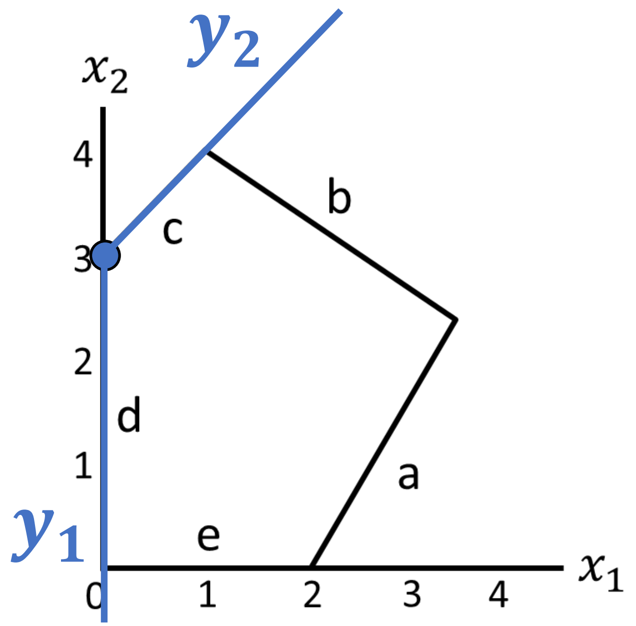
Stop at:

Vertex Local Coordinates:

Which constraint should we relax?

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)

Increasing y_2 worsens objective. Therefore, we should increase y_1, which is constraint (d).



```
Simplified_Simplex(LP)
   v = origin
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      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i – a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: c,d

Objective Value: 15

Relax: d

Stop at:

Vertex Local Coordinates:

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
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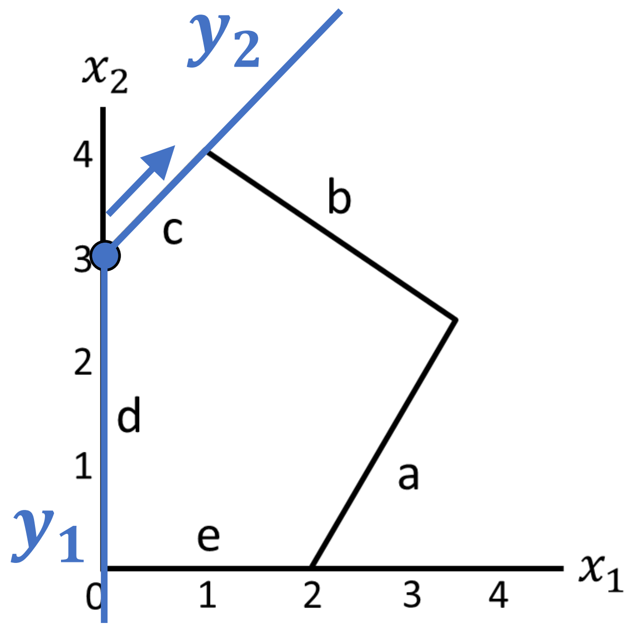
	$-y_1 + y_2 \leq 3$	(e)



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Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: c,d

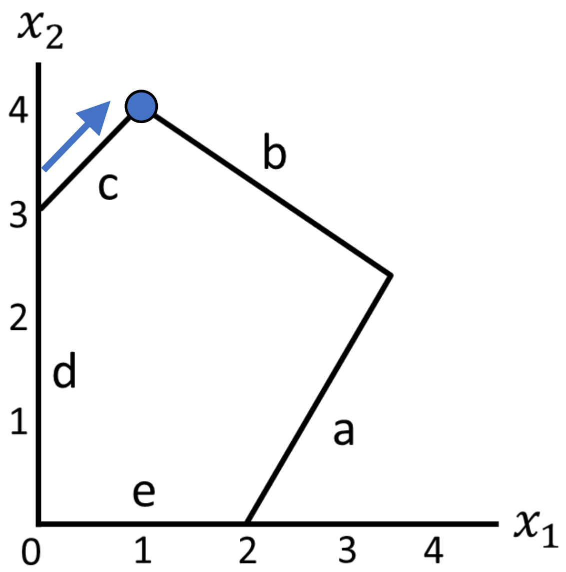
Objective Value: 15

Relax: d

Stop at: b,c

Vertex Local Coordinates:

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



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   v = origin
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      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·x ≤ b_i
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Vertex: ~~c,d~~ b,c

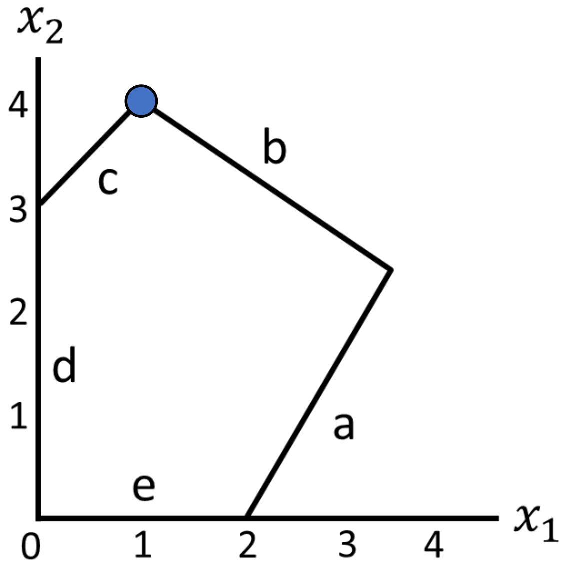
Objective Value: 15

Relax: d

Stop at: b,c

Vertex Local Coordinates:

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



```
Simplified_Simplex(LP)
   v = origin
   while cj > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint ai·y ≤ bi
         zi = bi – ai·y
      reformulate LP in terms of zi
   return v
```

Vertex: ~~c,d~~ b,c

Objective Value: 15

Relax: d

Stop at: b,c

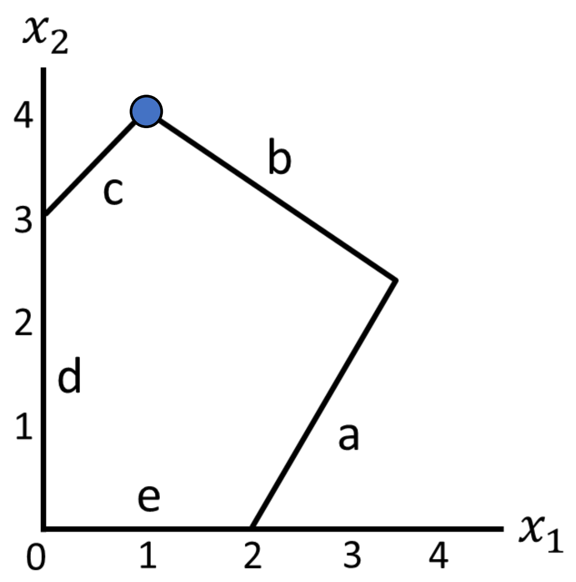
Vertex Local Coordinates:

$$3y_1 - 2y_2 \leq 3 \Longrightarrow z_1 = 3 - 3y_1 + 2y_2$$

$$y_2 \geq 0 \Longrightarrow z_2 = y_2$$

$$\Longrightarrow y_1 = 1 - \frac{1}{3}z_1 + \frac{2}{3}z_2, y_2 = z_2$$

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



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Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·y ≤ b_i
         z_i = b_i - a_i·y
      reformulate LP in terms of z_i
   return v
```

Vertex: ~~c,d~~ b,c

Objective Value: 15

Relax: d

Stop at: b,c

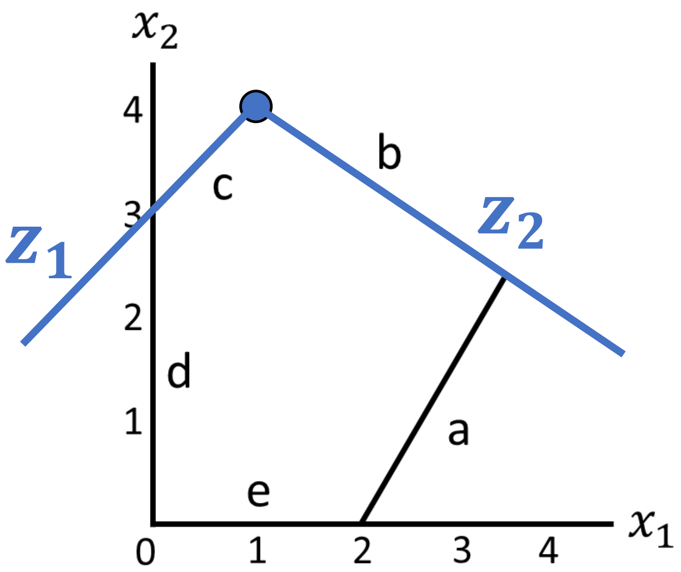
Vertex Local Coordinates:

$$3y_1 - 2y_2 \leq 3 \Longrightarrow z_1 = 3 - 3y_1 + 2y_2$$

$$y_2 \geq 0 \Longrightarrow z_2 = y_2$$

$$\Longrightarrow y_1 = 1 - \frac{1}{3}z_1 + \frac{2}{3}z_2, y_2 = z_2$$

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)



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   v = origin
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      stop at new constraint.
      v = new intersection.
      for constraint a_i·y ≤ b_i
         z_i = b_i - a_i·y
      reformulate LP in terms of z_i
   return v
```

Vertex: ~~c,d~~ b,c

Objective Value: 15

Relax: d

Stop at: b,c

Vertex Local Coordinates:

$$3y_1 - 2y_2 \le 3 \Rightarrow z_1 = 3 - 3y_1 + 2y_2$$

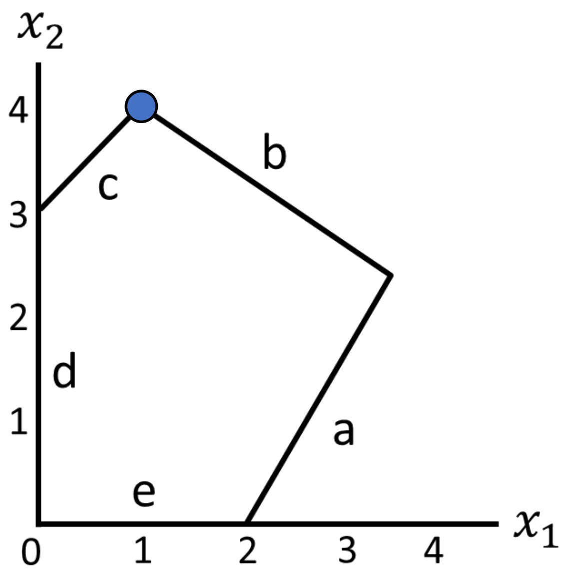
$$y_2 \ge 0 \Rightarrow z_2 = y_2$$

$$\Rightarrow y_1 = 1 - \frac{1}{3}z_1 + \frac{2}{3}z_2, y_2 = z_2$$

$$15 + 7y_1 - 5y_2 = 15 + 7 - \frac{7}{3}z_1 + \frac{14}{3}z_2 - 5z_2 = 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$$

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \le 7$	(a)
	$3y_1 - 2y_2 \le 3$	(b)
	$y_2 \ge 0$	(c)
	$y_1 \ge 0$	(d)
	$-y_1 + y_2 \le 3$	(e)

Objective:	$\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint a_i·y ≤ b_i
         z_i = b_i - a_i·y
      reformulate LP in terms of z_i
   return v
```

Vertex: ~~c,d~~ b,c

Objective Value: ~~15~~ 22

Relax: d

Stop at: b,c

Vertex Local Coordinates:

$$3y_1 - 2y_2 \leq 3 \Longrightarrow z_1 = 3 - 3y_1 + 2y_2$$

$$y_2 \geq 0 \Longrightarrow z_2 = y_2$$

$$\Longrightarrow y_1 = 1 - \frac{1}{3}z_1 + \frac{2}{3}z_2, y_2 = z_2$$

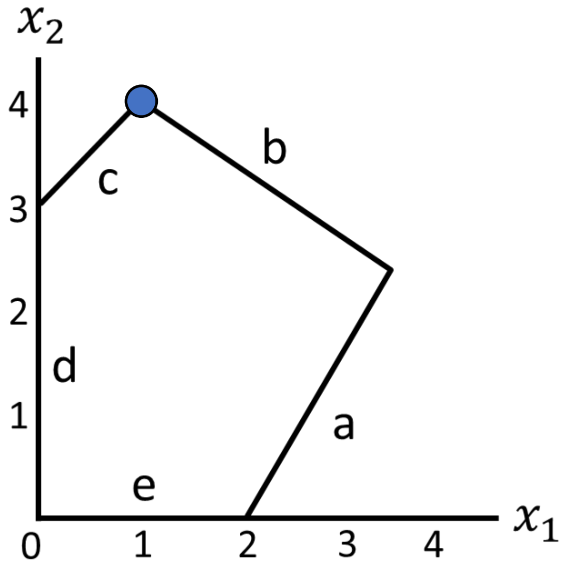
$$15 + 7y_1 - 5y_2 = 15 + 7 - \frac{7}{3}z_1 + \frac{14}{3}z_2 - 5z_2 = 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$$

Objective: $\max 15 + 7y_1 - 5y_2$

Subject to:
- $y_1 + y_2 \leq 7$ (a)
- $3y_1 - 2y_2 \leq 3$ (b)
- $y_2 \geq 0$ (c)
- $y_1 \geq 0$ (d)
- $-y_1 + y_2 \leq 3$ (e)

Objective: $\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$

Subject to:
- (a)
- (b)
- (c)
- (d)
- (e)



```
Simplified_Simplex(LP)
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      v = new intersection.
      for constraint a_i·y ≤ b_i
         z_i = b_i - a_i·y
      reformulate LP in terms of z_i
   return v
```

Vertex: ~~c,d~~ b,c

Objective Value: ~~15~~ 22

Relax: d

Stop at: b,c

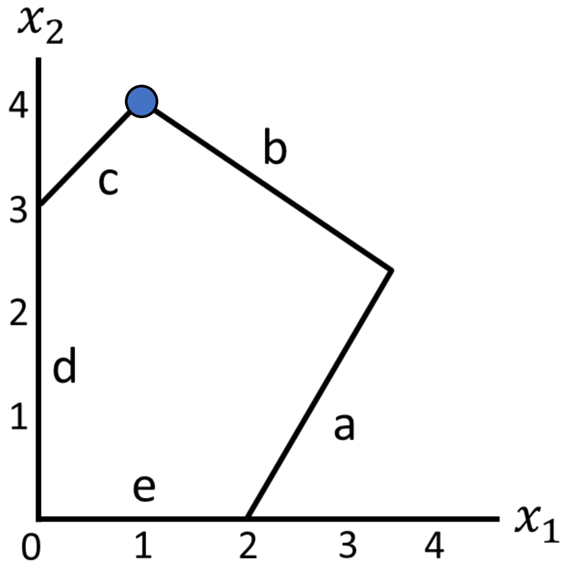
Vertex Local Coordinates:

$$3y_1 - 2y_2 \leq 3 \Longrightarrow z_1 = 3 - 3y_1 + 2y_2$$
$$y_2 \geq 0 \Longrightarrow z_2 = y_2$$
$$\Longrightarrow y_1 = 1 - \frac{1}{3}z_1 + \frac{2}{3}z_2, y_2 = z_2$$

$$15 + 7y_1 - 5y_2 = 15 + 7 - \frac{7}{3}z_1 + \frac{14}{3}z_2 - 5z_2 = 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$$

Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)

Objective:	$\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



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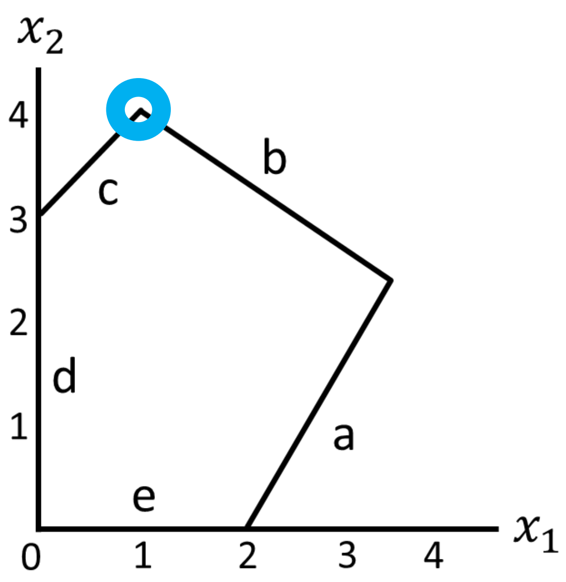
Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \le 7$	(a)
	$3y_1 - 2y_2 \le 3$	(b)
	$y_2 \ge 0$	(c)
	$y_1 \ge 0$	(d)
	$-y_1 + y_2 \le 3$	(e)

Objective:	$\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



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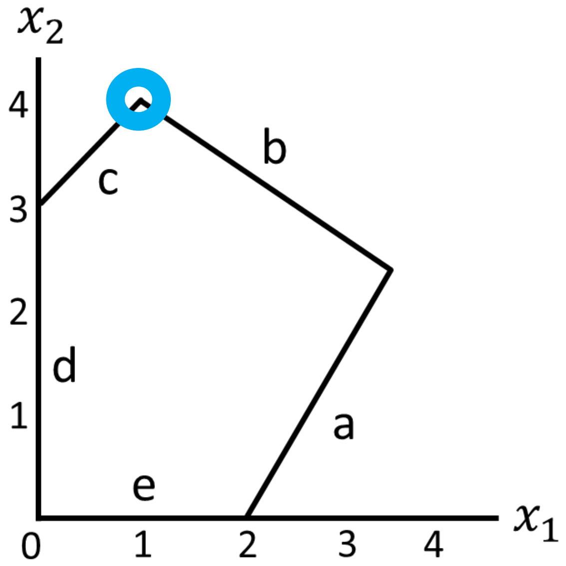
Objective:	$\max 15 + 7y_1 - 5y_2$	
Subject to:	$y_1 + y_2 \leq 7$	(a)
	$3y_1 - 2y_2 \leq 3$	(b)
	$y_2 \geq 0$	(c)
	$y_1 \geq 0$	(d)
	$-y_1 + y_2 \leq 3$	(e)

⇒

Objective:	$\max 22 - \frac{7}{3}z_1 - \frac{1}{3}z_2$	
Subject to:		(a)
		(b)
		(c)
		(d)
		(e)



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         z_i = b_i - a_i·y
      reformulate LP in terms of z_i
   return v
```

Loose Ends:

1. Starting Vertex
2. Unbounded/infeasible solution
3. Degenerate vertices
4. Running Time

Objective: $\max 15 + 7y_1 - 5y_2$

Subject to:

$y_1 + y_2 \leq 7$ (a)

$3y_1 - 2y_2 \leq 3$ (b)

$y_2 \geq 0$ (c)

$y_1 \geq 0$ (d)

$-y_1 + y_2 \leq 3$ (e)

Objective: $\max 22 - \frac{7}{3} z_1 - \frac{1}{3} z_2$

Subject to:

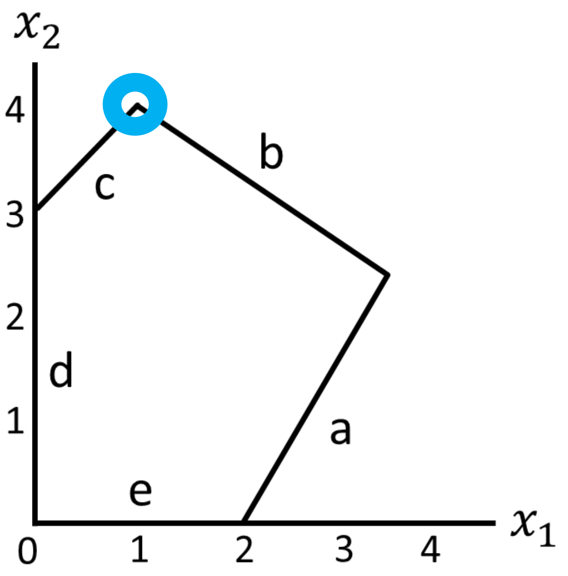
(a)

(b)

(c)

(d)

(e)



```
Simplified_Simplex(LP)
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         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Running Time ($n =$ # variables, $m =$ # constraints defined by A):

```
Simplified_Simplex(LP)
   v = origin
   while cj > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint ai·x ≤ bi
         yi = bi - ai·x
      reformulate LP in terms of yi
   return v
```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Running Time ($n = \#$ variables, $m = \#$ constraints defined by A):

- Number of possible neighbors for a given vertex?

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Running Time (n = # variables, m = # constraints defined by A):

- Number of possible neighbors for a given vertex?

n non-negativity constraints are forming the vertex

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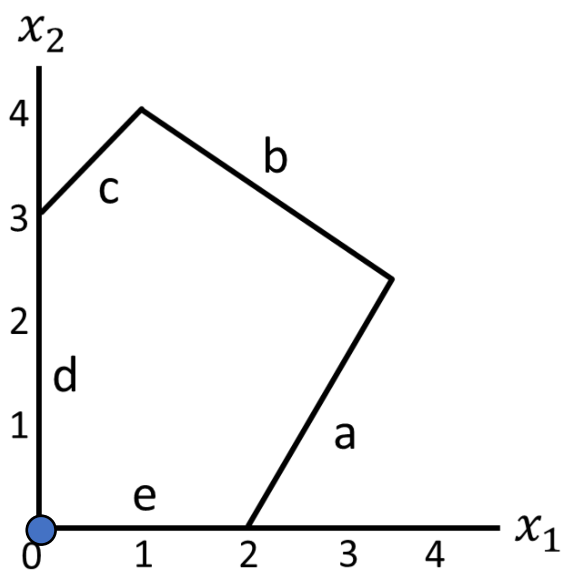

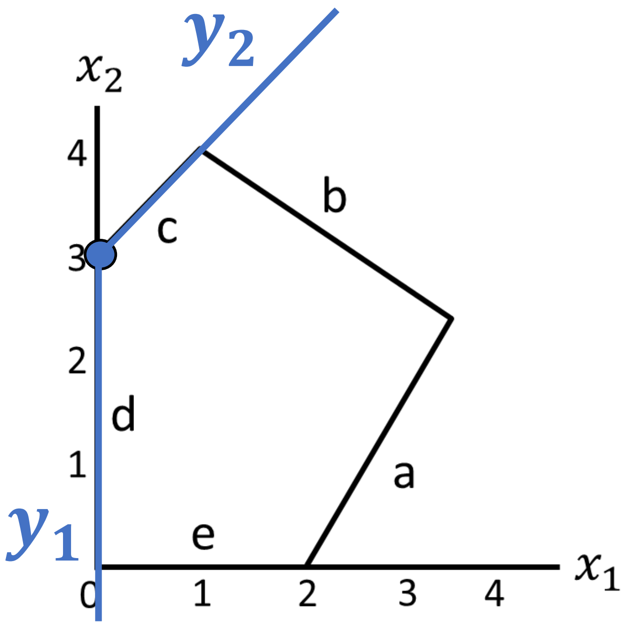

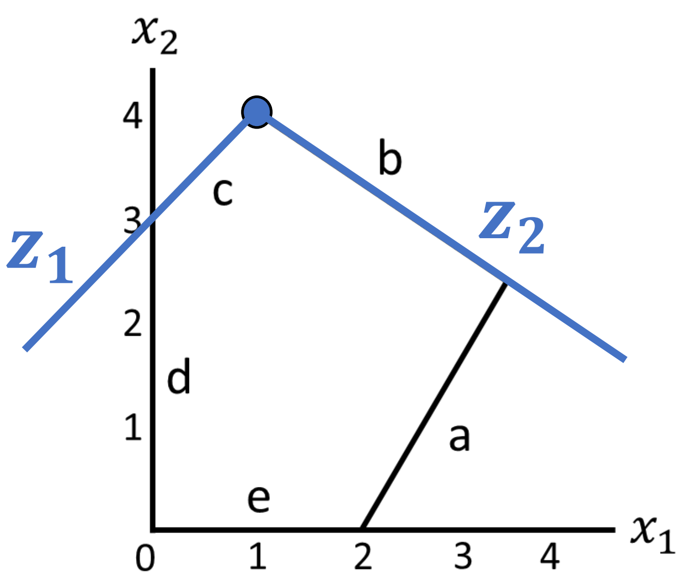

Objective: $\max 2x_1 + 5x_2$

Subject to:

$2x_1 - x_2 \leq 4$ (a)

$x_1 + 2x_2 \leq 9$ (b)

$-x_1 + x_2 \leq 3$ (c)

$x_1 \geq 0$ (d)

$x_2 \geq 0$ (e)

Objective: $\max 15 + 7y_1 - 5y_2$

Subject to:

$y_1 + y_2 \leq 7$ (a)

$3y_1 - 2y_2 \leq 3$ (b)

$y_2 \geq 0$ (c)

$y_1 \geq 0$ (d)

$-y_1 + y_2 \leq 3$ (e)

```
Simplified_Simplex(LP)
   v = origin
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      v = new intersection.
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         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```

Objective:	$\max c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Running Time (n = # variables, m = # constraints defined by A):

- Number of possible neighbors for a given vertex?

n non-negativity constraints are forming the vertex, we will replace one of them with one of the 'regular' constraints.

```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
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Objective: max $c^T x$
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Running Time (n = # variables, m = # constraints defined by A):

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n non-negativity constraints are forming the vertex, we will replace one of them with one of the 'regular' constraints. Thus, nm options for neighboring vertices

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   v = origin
   while c_j > 0 for some j
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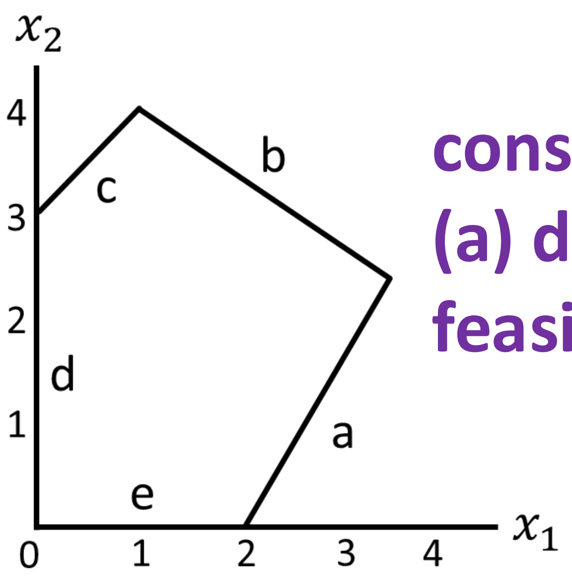
         y_i = b_i - a_i·x
      reformulate LP in terms of y_i
   return v
```



constraints (c) and (a) do not form a feasible vertex.

Running Time (n = # variables, m = # constraints defined by A):

- Number of possible neighbors for a given vertex?

n non-negativity constraints are forming the vertex, we will replace one of them with one of the 'regular' constraints. Thus, nm options for neighboring vertices, but not all combinations are feasible.

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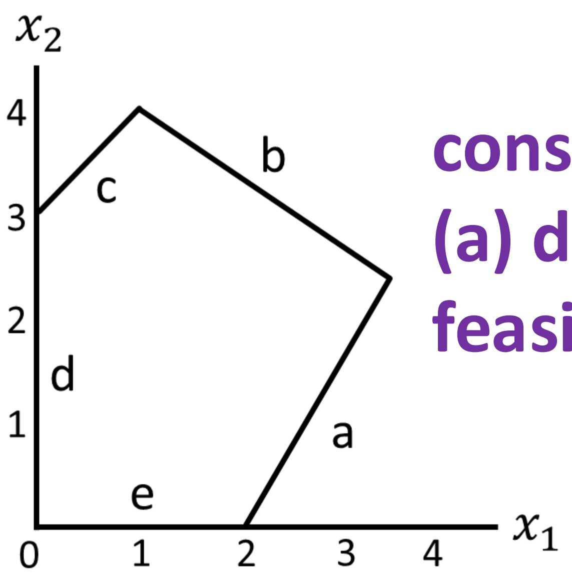

constraints (c) and (a) do not form a feasible vertex.

Running Time (n = # variables, m = # constraints defined by A):

- Number of possible neighbors for a given vertex ∈ $O(nm)$

n non-negativity constraints are forming the vertex, we will replace one of them with one of the 'regular' constraints. Thus, nm options for neighboring vertices, but not all combinations are feasible.

```
Simplified_Simplex(LP)
   v = origin
   while c_j > 0 for some j
      relax tight constraint.
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      reformulate LP in terms of y_i
   return v
```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

Running Time ($n =$ # variables, $m =$ # constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
- Testing if possible neighbor is feasible $\in O(n^3)$

```
Simplified_Simplex(LP)
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      reformulate LP in terms of y_i
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```

Objective: $\max c^T x$
Subject to: $A x \leq b$
$x \geq 0$

Running Time ($n =$ # variables, $m =$ # constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
- Testing if possible neighbor is feasible $\in O(n^3)$

$\Rightarrow O(n^4 m)$ per iteration

```
Simplified_Simplex(LP)
   v = origin
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      stop at new constraint.
      v = new intersection.
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         yi = bi – ai·x
      reformulate LP in terms of yi
   return v
```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
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Running Time ($n =$ # variables, $m =$ # constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
- Testing if possible neighbor is feasible $\in O(n^3)$

$\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ **Actually $O(nm)$**

Since we don't consider all possible neighbors and swapping coordinate systems (rewriting the LP) can happen in $O((m+n)n)$.

```
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   v = origin
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Objective:	$\max c^T x$
Subject to:	$A x \leq b$
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Running Time ($n = \#$ variables, $m = \#$ constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
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  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$

- Possible number of iterations?

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  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$

- Possible number of iterations?

Possible number of feasible vertices

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   v = origin
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```

Objective:	max $c^T x$
Subject to:	$A x \leq b$
	$x \geq 0$

$m + n$ constraints in total.

Running Time ($n = \#$ variables, $m = \#$ constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
- Testing if possible neighbor is feasible $\in O(n^3)$

  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$
- Possible number of iterations $= \binom{m+n}{n}$

Possible number of feasible vertices

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   v = origin
   while cj > 0 for some j
      relax tight constraint.
      stop at new constraint.
      v = new intersection.
      for constraint ai·x ≤ bi
         yi =
      reformu
   return v
```

Objective: max $c^T x$

Subject to: $A x \leq b$

$x \geq 0$

nts

$$\binom{m+n}{n} \geq \binom{2n}{n} = \frac{(2n)!}{n!n!} = \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)\ldots}{n(n-1)(n-2)\ldots n(n-1)(n-2)\ldots}$$

$$= \frac{2\not{n}(2n-1)2\not{(n-1)}(2n-3)2\not{(n-2)}\ldots}{\not{n}\not{(n-1)}\not{(n-2)}\ldots n(n-1)(n-2)\ldots}$$

$$\geq \frac{2^n(2n-1)(2n-3)\ldots}{n!} \geq 2^n$$

Runnin

-
- Testing if possible neighbor is feasible $\in O(n^3)$

  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$
- Possible number of iterations $= \binom{m+n}{n}$

Possible number of feasible vertices

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Objective: max $c^T x$
Subject to: $A x \leq b$
$x \geq 0$

$$\binom{m+n}{n} \geq \binom{2n}{n} = \frac{(2n)!}{n!n!} = \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)\ldots}{n(n-1)(n-2)\ldots n(n-1)(n-2)\ldots}$$

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nts

Runnin

- Testing if possible neighbor is feasible $\in O(n^3)$

  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$
- Possible number of iterations $= \binom{m+n}{n} \in \Omega(2^n)$

Possible number of feasible vertices

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      for cor
         yi =
      reformu ate LP in terms of yi
   return v
```

Obj

$\max c^T x$

$A x \leq b$

$x \geq 0$

Simplex is Exponential!

Running Time (n = # variables, m = # constraints defined by A):

- Number of possible neighbors for a given vertex $\in O(nm)$
- Testing if possible neighbor is feasible $\in O(n^3)$

  $\Rightarrow$ ~~$O(n^4 m)$ per iteration~~ Actually $O(nm)$
- Possible number of iterations $= \binom{m+n}{n} \in \Omega(2^n)$

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```

Obj

$\max c^T x$

$A x \leq b$

$x \geq 0$

Simplex is Exponential!

History Lesson:

- Solving systems of linear inequalities dates back to the 1800’s.
- Linear programming was widely studied in the 1940’s.
- Simplex invented in 1947.
- Specific LPs that Simplex takes exponential time on discovered in 1972.
- Soviet mathematician found polynomial time algorithm in 1979.
- Interior point method found in 1984.
- CPLEX released in 1988.